Propagation Constants of a Waveguide Containing Parallel Sheets of Finite Conductivity

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Abstract—**The propagation constants of a rectangular waveguide containing periodic, parallel sheets having finite conductivity were determined using an iterative computer program. The dispersion equation was found in a matrix formulation which was conducive to computer solution. This equation was solved for various values of conductivity and sheet spacing. Comparisons were made with the propagation constants found assuming an infinite array of thin parallel resistive sheets in free space for the case where the direction of propagation and the electric field vector were both parallel to the sheets. The propagation constants for the infinite array case have been determined by both conformal mapping techniques for certain limiting conditions and by a computer solution of the dispersion equation for sheet spacings and conductivities of interest. The results should prove useful for the design of absorbing elements and terminations for waveguides.**

I. Introduction

FOR PURPOSES of analysis, the waveguide was assumed to be rectangular with perfectly conducting walls and attenuation provided by an array of resistive sheets of finite thickness parallel to the side walls. Using the transverse resonance method, the dispersion equation was written in a matrix formulation suitable for an iterative computer program. Computer solutions were obtained for several representative values of bulk conductivity and different quantities of sheets in a 30 cm wide waveguide. Converting the lossy sheets to the alternate thin-sheet representation of ohms per unit square permitted direct comparison with the infinite array representation.

Computer solutions to the infinite array of parallel resistive sheets were also obtained both to check the range of validity of the approximate solutions of Suetake and Griemsmann [1] as well as to provide a comparison with the waveguide approximation.

It is shown that when the frequency is 1.5 to 2 times the TE_{10} mode cutoff frequency in the unloaded waveguide and the number of sheets exceeds 3 or 4, the loaded waveguide solutions closely approach the infinite array solutions. In physical terms, the conducting boundary is decoupled by the resistive sheets. Thus, an attenuating grid can be designed using the infinite array theory regardless of the cross section of the duct—subject only to the limitation that the actual grid contain at least 3 sheets.

II. Model Configuration

The propagation constants for a single dielectric slab in a rectangular waveguide [2]–[6], and the expected microwave attenuation have been studied previously. A means for determining the propagation constants of several parallel sheets having finite conductivity in a rectangular waveguide are discussed here. These results are compared to the propagation constant and the impedance found for the fundamental TE mode of a plane wave propagating through an infinite array of parallel resistive sheets [1] where the direction of propagation and the electric field vector were both parallel to the sheets. In both media, the slab-loaded waveguide and the infinite resistive sheet array, the transcendental equation was found through the use of a transverse resonance procedure. For the infinite array case the dispersion equation has been solved by conformal mapping techniques [1]. In addition, a number of solutions of practical interest were obtained through a computer solution of the dispersion equation. The computer results provided more data than that obtained by the conformal mapping solutions.

In the analysis presented in this paper a transverse resonance procedure is used to examine the propagation characteristics of a waveguide containing an arbitrary number of uniformly spaced parallel sheets having finite conductivity. The lossy sheets are parallel to two walls of the waveguide and the outer walls are all assumed to be perfectly conducting. The results of the two approaches are compared and for sheet spacing and conductivity values of interest, the results are quite similar.

III. Analytical Approach

The model which was studied is shown in Fig. 1. There is an arbitrary number of parallel resistive sheets of equal spacing within a rectangular waveguide. A hybrid of the usual E- and H-type modes is required for this problem and it is convenient to classify the modes as longitudinal section electric (LSE) or longitudinal section magnetic (LSM). The LSE modes are those for which the electric field is confined to longitudinal planes parallel to the resistive sheets. For LSM modes the magnetic field is confined to planes parallel to the resistive sheets.

There are a number of approaches to the problem. First, Maxwell's equations can be solved completely in each

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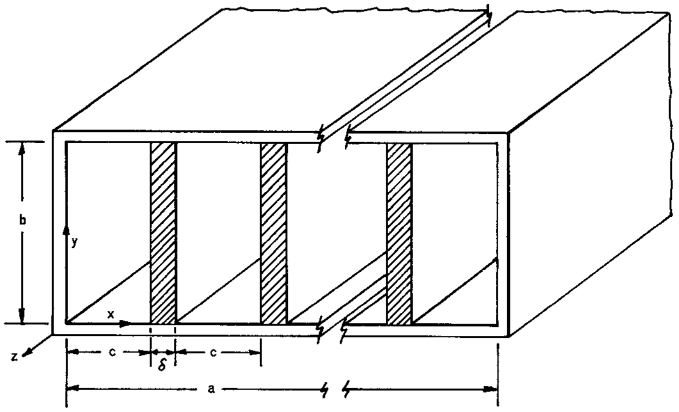


Fig. 1. Waveguide with parallel absorbing sheets.

homogeneous region separately; that is, in each resistive sheet and each air space, and the fields matched at the various boundaries. This procedure can always be carried out, but it becomes extremely cumbersome for more than one or two sheets. In addition, the entire process must be repeated if the number of sheets is changed.

If the sheets are spaced periodically across the guide, then a second method can be attempted. The conductivity is a periodic function of x and may be expanded into a Fourier series which, when substituted into the wave equation, yields a form that applies everywhere within the guide. However, the overall medium is now inhomogeneous and the separation of variables leads to Hill's equation involving the coordinate x. Formal solutions may be written down and the boundary conditions applied, but the dispersion equation takes the form of an infinite-by-infinite determinant. Approximate solutions for the allowed propagation constants may be obtained only under quite restrictive assumptions.

A number of perturbation and variational techniques suggest themselves, but it is found that either they are accurate only for very limited ranges of parameter values or else they are much too cumbersome to be useful. The transverse resonance method, *while not yielding all details of the solution*, does supply the propagation constants which are all that are desired in this analysis. The procedure involves evaluation of transverse impedances in each region and impedance matching at the interfaces between the regions. This method is exact, can accommodate any number of resistive sheets, and lends itself to computer solution.

Development of Working Equations

1) *LSE Modes:* Figure 1 depicts the model chosen. The parameter c is the space between sheets and between the end sheets and the walls. The permeability μ, permittivity ϵ, conductivity σ, and wavenumber k are

$$\mu = \mu_0,\ \epsilon = \epsilon_0,\ \sigma = 0 \quad k^2 = k_0^2 = \omega^2\mu_0\epsilon_0$$

between the sheets

$$\mu = \mu_0,\ \epsilon = \kappa\epsilon_0,\ \sigma = \sigma_0 \quad k^2 = k_r^2 = \omega^2\mu_0\epsilon_0\left(\kappa - j\frac{\sigma_0}{\omega\epsilon_0}\right)$$

within the sheets. The electric field must satisfy the wave equation

$$\nabla \times \nabla \times E - k^2 E = 0 \tag{1}$$

where the value of k appropriate to the region is inserted.

The working equations for LSE modes will be found first. Since the structure is uniform along the z-dimension, the z-variations must be of the form $e^{-\gamma z}$. For LSE modes the field components are [2], [3]

$$\begin{aligned} E_x &= 0 & H_x &= \left[\frac{\partial^2\psi}{\partial x^2} + k^2\psi\right]e^{-\gamma z} \\ E_y &= jkZ\gamma\psi e^{-\gamma z} & H_y &= \frac{\partial^2\psi}{\partial x \partial y}e^{-\gamma z} \\ E_z &= jkZ\frac{\partial\psi}{\partial y}e^{-\gamma z} & H_z &= -\gamma\frac{\partial\psi}{\partial x}e^{-\gamma z}. \end{aligned} \tag{2}$$

The potential function ψ satisfies

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + k_c^2\psi = 0 \tag{3}$$

where $k_c^2 = \gamma^2 + k^2$ and k_c and k are constants within a given region. In (2) Z and k are given by

$$Z = Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}; \quad k = k_0$$

between the sheets

$$Z = Z_r = \sqrt{\frac{\mu_0}{\epsilon_0\left(\kappa - j\dfrac{\sigma_0}{\omega\epsilon_0}\right)}}; \quad k = k_r$$

within the sheets.

Separating variables by letting $\psi = X(x)Y(y)$, we obtain

$$\begin{aligned} \frac{d^2Y}{dy^2} + k_y^2 Y &= 0 \\ \frac{d^2X}{dx^2} + k_x^2 X &= 0. \end{aligned} \tag{4}$$

Solving the differential equation in y and applying the boundary conditions for the perfectly conducting waveguide walls at $y = 0$, b gives

$$Y_n = A_n \cos k_{yn} y \tag{5}$$

where

$$k_{yn} = \frac{n\pi}{b}, \qquad n \text{ integer}.$$

The separation constant k_y is independent of location.

Solutions for the differential equation in x may be written as a superposition of incident and reflected waves

$$X = e^{-jk_x x} + \mathrm{R}e^{jk_x x} = [1 + \mathrm{R}e^{2jk_x x}]e^{-jk_x x} \tag{6}$$

where R is a transverse reflection coefficient. The separation constant k_x has one value in air and another in the sheets.

Utilizing the results of (2), (5), and (6) the electric field vectors can be expressed as

$$E_y = E_y^+ + E_y^- = G_y[1 + Re^{2jk_x x}]e^{-jk_x x} \quad (7)$$

$$E_z = E_z^+ + E_z^- = G_z[1 + Re^{2jk_x x}]e^{-jk_x x} \quad (8)$$

where $G_y(y, z) = jkZ\gamma Ye^{-\gamma z}$ and $G_z(y, z) = jkZ(dY/dy)e^{-\gamma z}$. The + and − superscripts denote the portions of the solutions that vary as $e^{-jk_x x}$ and $e^{jk_x x}$, respectively.

We define a $+x$ directed impedance as

$$Z_x = \frac{E_y^+}{H_z^+} = -\frac{E_z^+}{H_y^+} \quad \text{or} \quad Z_x = -\frac{E_y^-}{H_z^-} = \frac{E_z^-}{H_y^-}. \quad (9)$$

Application of (2) reveals that

$$Z_x = \frac{kZ}{k_x}.$$

The fields parallel to the longitudinal sheets are

$$\boldsymbol{E}_l = \boldsymbol{a}_y[E_y^+ + E_y^-] + \boldsymbol{a}_z[E_z^+ + E_z^-]$$

$$\boldsymbol{H}_l = \boldsymbol{a}_y[H_y^+ + H_y^-] + \boldsymbol{a}_z[H_z^+ + H_z^-]$$

and these may be transformed into

$$\boldsymbol{E}_l = V(x)[\boldsymbol{a}_y G_y + \boldsymbol{a}_z G_z]$$

$$\boldsymbol{H}_l = I(x)[-\boldsymbol{a}_y G_z + \boldsymbol{a}_z G_y]$$

through use of (7), (8), and (9). The voltage $V(x)$ and the current $I(x)$ are defined by

$$V(x) = [1 + Re^{2jk_x x}]e^{-jk_x x} \quad (10)$$

$$I(x) = [1 - Re^{2jk_x x}]\frac{e^{-jk_x x}}{Z_x}. \quad (11)$$

At some new location $(x+d)$, the voltage and current are

$$V(x + d) = [1 + Re^{2jk_x(x+d)}]e^{-jk_x(x+d)} \quad (12)$$

$$I(x + d) = [1 - Re^{2jk_x(x+d)}]\frac{e^{-jk_x(x+d)}}{Z_x}. \quad (13)$$

The voltage and current at one location may be expressed in terms of the voltage and current at another location in the same medium as follows:

$$V(x) = m_{11}V(x + d) + m_{12}I(x + d)$$

$$I(x) = m_{21}V(x + d) + m_{22}I(x + d).$$

In matrix notation these equations become

$$\begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = [M]\begin{bmatrix} V(x+d) \\ I(x+d) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} V(x+d) \\ I(x+d) \end{bmatrix}.$$

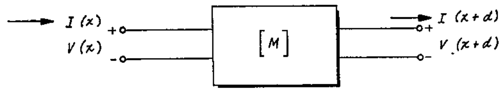


Fig. 2. Network representation of medium.

Solving (10)–(13) for the coefficients yields

$$m_{11} = m_{22} = \cos k_x d$$

$$m_{12} = jZ_x \sin k_x d$$

$$m_{21} = \frac{j \sin k_x d}{Z_x}.$$

The transformations correspond to a representation of the medium by a two-port network in the x-dimension as depicted in Fig. 2. At a boundary between two different media the tangential electric and magnetic fields must be continuous, hence the voltage and current as defined here must be continuous. Transforming across a boundary is equivalent to connecting two networks in tandem.

Matrices $[M_1]$ and $[M_2]$ are the matrix $[M]$ evaluated for appropriate distances in the first and second media, respectively. Then

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [M_1]\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = [M_2]\begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

which combine to give

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [M_1][M_2]\begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = [P]\begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

where $[P] = [M_1][M_2]$. This procedure may be carried through as many sections as desired.

Suppose there are N identical resistive sheets in the guide. There are then $2N+1$ regions and $2N+1$ matrices. The transformation from $x = o$ to $x = a$ then becomes

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [M_1][M_2] \cdots [M_{2N+1}]\begin{bmatrix} V_{2N+2} \\ I_{2N+2} \end{bmatrix} = [M^0]\begin{bmatrix} V_{2N+2} \\ I_{2N+1} \end{bmatrix}$$

where

$$[M^0] = \begin{bmatrix} m_{11}^0 & m_{12}^0 \\ m_{21}^0 & m_{22}^0 \end{bmatrix} = [M_1][M_2] \cdots [M_{2N+1}].$$

For our model all odd matrices are identical and all even matrices are identical. Let $[M_1]$ represent the matrix for a free space section of width c and $[M_2]$ the matrix for a resistive sheet of width δ. The elements for these matrices are

$$[M_1] = \begin{bmatrix} \cos k_{x0}c & jZ_{x0} \sin k_{x0}c \\ \dfrac{j \sin k_{x0}c}{Z_{x0}} & \cos k_{x0}c \end{bmatrix} \quad (14)$$

$$[M_2] = \begin{bmatrix} \cos k_{xr}\delta & jZ_{xr} \sin k_{xr}\delta \\ \dfrac{j \sin k_{xr}\delta}{Z_{xr}} & \cos k_{xr}\delta \end{bmatrix} \quad (15)$$

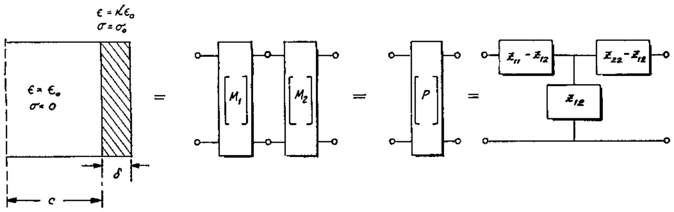


Fig. 3. Unit cell equivalences.

where k_{x0} is an unknown to be determined, and

$$k_{xr}^2 = k_{x0}^2 + (\kappa - 1)k_0^2 - j\omega\mu_0\sigma_0$$

$$Z_{x0} = \frac{\omega\mu_0}{k_{x0}} \quad \text{and} \quad Z_{xr} = \frac{\omega\mu_0}{k_{xr}}.$$

An adjacent air space and resistive sheet can be represented as an equivalent cell as shown in Fig. 3. Other equivalent cells are possible but this one is most convenient for this application since it involves only two adjacent regions. Then for N unit cells in tandem plus one air space

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [P]^N[M_1]\begin{bmatrix} V_{2N+2} \\ I_{2N+2} \end{bmatrix} = [M^0]\begin{bmatrix} V_{2N+2} \\ I_{2N+2} \end{bmatrix} \quad (16)$$

and

$$[M^0] = \begin{bmatrix} m_{11}^0 & m_{12}^0 \\ m_{21}^0 & m_{22}^0 \end{bmatrix} = [P]^N[M_1]. \quad (17)$$

From (16) and (17)

$$V_1 = m_{11}^0 V_{2N+2} + m_{12}^0 I_{2N+2} \quad (18)$$

$$I_1 = m_{21}^0 V_{2N+2} + m_{22}^0 I_{2N+2}. \quad (19)$$

The dispersion relationships can be stated in a general sense in terms of transverse impedances at $x=0$ and $x=a$. If we define $Z_L = V_1/I_1$ and $Z_R = V_{2N+2}/I_{2N+2}$, substitute into (18) and (19) and equate the determinant to zero, we obtain the dispersion equation

$$[m_{21}^0 Z_R + m_{22}^0]Z_L = m_{11}^0 Z_R + m_{12}^0.$$

If, however, the terminations are symmetric as in a waveguide, we recognize two mode types. For one type $\boldsymbol{E}_l$ is an even distribution about $x=a/2$ and for the other type $\boldsymbol{E}_l$ is an odd distribution about $x=a/2$. Then

$$V_{2N+2} = \pm V_1;\; I_{2N+2} = \mp I \quad \text{upper sign for even modes, lower sign for odd modes}$$

and using these in (18) and (19) gives two expressions

$$Z_L = \frac{\mp m_{12}^0}{1 \mp m_{11}^0} \quad (20)$$

$$Z_L = \frac{1 \pm m_{22}^0}{\pm m_{21}^0}. \quad (21)$$

For a waveguide with perfectly conducting walls $\boldsymbol{E}_l = 0$ at $x=0$ and $x=a$, thus $Z_L=0$. For this particular case, (20) reduces to the dispersion equation

$$m_{12}^0 = 0$$

which yields all LSE modes. The alternate form

$$m_{22}^0 = \mp 1$$

is obtained from (21). Even modes are given by $m_{22}^0 = -1$ and odd modes by $m_{22}^0 = +1$. Multiplication of the matrices to obtain explicit formulas for m_{12}^0 or m_{22}^0 in terms of k_{x0} need not be written out as this can be included in the computer program as an iterative process.

2) *LSM Modes:* The development for LSM modes is identical with that for LSE modes. The only difference is in the expression for the x-directed impedance which for LSM modes becomes

$$Z_x = \frac{Zk_x}{k} \quad (22)$$

The dispersion equation for LSE modes applies to LSM modes if the LSE impedance is replaced by the LSM impedance given in (22).

IV. Computer Results

A digital computer was used to find the propagation constants for the waveguide containing conductive sheets. The procedure involved assuming an initial value for k_{x0} and then making successive corrections to k_{x0} to make m_{12}^0 or $m_{22}^0 \mp 1$ approach zero. The estimation procedure requires careful attention to ensure that solutions for the desired mode are obtained. A similar procedure was used to solve the dispersion equation for the infinite array of sheets as presented in Appendix A.

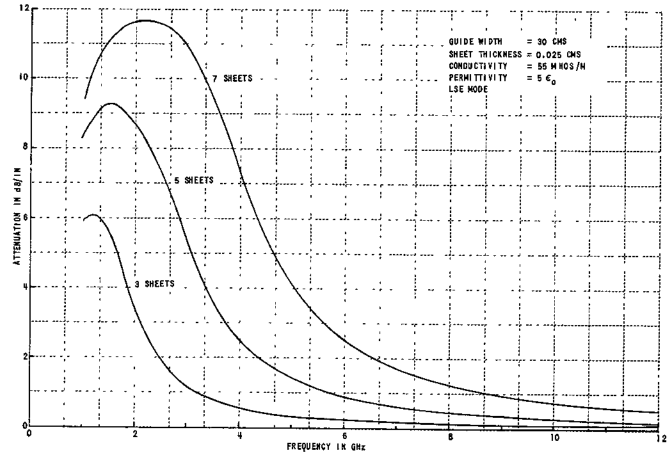
Practical considerations dictated the choice of parameter values. Computations were made for conductive sheets of thickness 0.025 cm, and for waveguides of 30 cm and 40 cm width. Frequencies from 1 to 12 GHz and conductivities from 5 to 55 mhos/m were considered. This frequency range excludes the cutoff region for the equivalent lossless waveguides. The permittivity of the lossy material was assumed to be $5\epsilon_0$, although it was found that for the thin sheets of interest here the attenuation is only very slightly dependent upon permittivity. In actual lossy materials the conductivity and permittivity are frequency sensitive, however, such variations with frequency were not included in this program.

Most data were obtained for lowest-order LSE mode since it is the dominant mode of the structure. Some results for an LSM mode were found for comparison purposes. A mode numbering system compatible with the numbering system for the waveguide with the sheets removed cannot be established. This occurs because of the mode dependence upon conductivity. For low conductivity the lowest-order LSE mode corresponds closely to the LSE_{10} mode of an empty waveguide. As conductivity is increased, the sheets tend to decouple the sections of the guide and the lowest-order LSE mode approaches the $LSE_{N+1,0}$ mode of the empty waveguide, where N denotes the number of sheets.

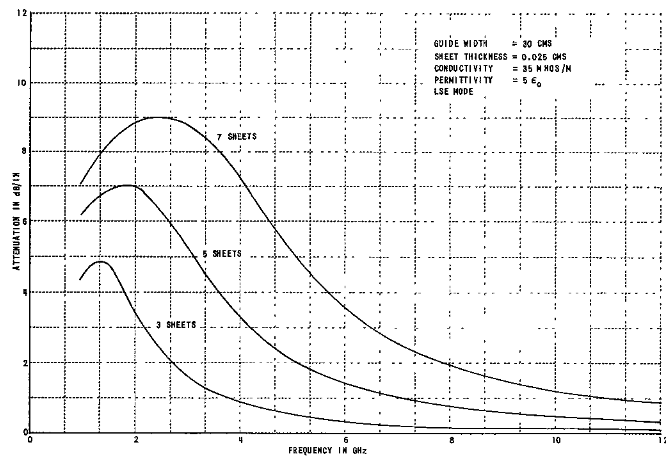
The attentuation of a wave propagating in the LSE mode down a waveguide for various values of conductivity are shown on Fig. 4(a), (b), and (c). The values are shown for three, five, and seven conductive sheets uniformly spaced across a waveguide of 30 cm width. The curves at each value of conductivity show attenuation peaks of increasing amplitude and greater width in frequency as the number of sheets is increased. The increase in attenuation with number of sheets is simply a result of the guide containing a larger amount of resistive material. For the dimensions and conductivities considered $\frac{1}{2}\sigma\sqrt{\mu/\epsilon}>k_c$. Thus, in a waveguide completely filled with conductive material (as discussed in Appendix B) the attenuation is a monotonically increasing function of frequency and approaches $\alpha_\infty=\frac{1}{2}\sigma\sqrt{\mu/\epsilon}$ as frequency approaches infinity. This represents the maximum attenuation that can be achieved as the number of sheets is increased. The three cases shown have peaks much below this maximum; however, this limit indicates that the peaks must become higher, shift toward higher frequencies and become broader as the number of sheets is increased. The attenuation curves also show a decrease and broadening of the peaks with decreasing conductivity. For sufficiently low conductivities the attenuation would become a monotonically decreasing function of frequency above the cutoff frequency, since a completely filled waveguide exhibits a monotonically decreasing attenuation with frequency for $\frac{1}{2}\sigma\sqrt{\mu/\epsilon}<k_c$.

At sufficiently low frequencies the resistive sheets act much like good conductors and tend to decouple the fields in the regions separated by the sheets. Losses are low, but attenuation due to the cutoff characteristics of the guide may be high. At sufficiently high frequencies the sheets behave as good dielectrics and losses are again low. At some intermediate frequency attenuation due to the resistive sheets will be a maximum and the curve peaks. For a completely filled guide this peak is at infinity whereas for partially filled guides it occurs at a lower frequency. The amplitude of the peak decreases with conductivity and for low conductivities there may not be a peak above the cutoff frequency of the equivalent lossless guide.

Attenuation versus frequency for constant sheet spacing but different numbers of sheets is shown in Fig. 5(a), (b), and (c). The three cases are for a spacing of 5 cm in a guide of 30 cm width, 40 cm width, and for an infinite array of sheets. The latter case was solved using the dispersion equation of Suetake and Griemsmann [1]. For $k\geq 2k_c$ the curves for different numbers of sheets show remarkable agreement. Such agreement will occur, however, only if $\delta/c\ll 1$. The results indicate that for five or more sheets the side walls have very little effect on the attenuation. The interaction between the fields and the lossy sheets is determined by the sheet configuration rather than the side wall. Regardless of the shape of the external walls or whether they are present, the attenuation level remains essentially unchanged. The shape of the waveguide will not influence the level of attenuation attainable as long as the waveguide cutoff frequency of the waveguide is at most 60 percent of the operating frequency. The attenuating material can be placed in the most advantageous position in the waveguide.



(a)



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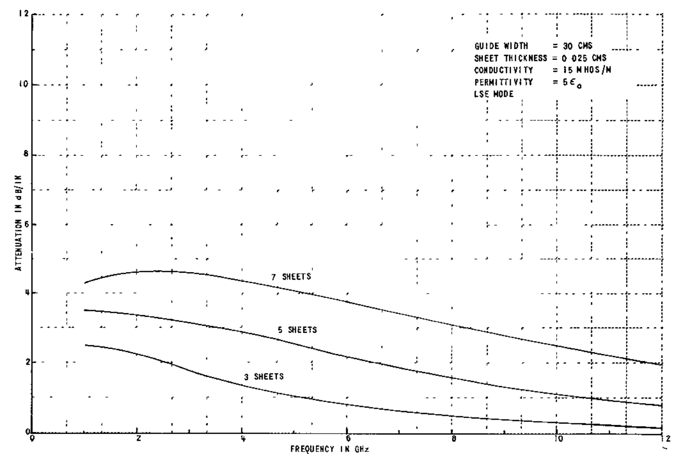


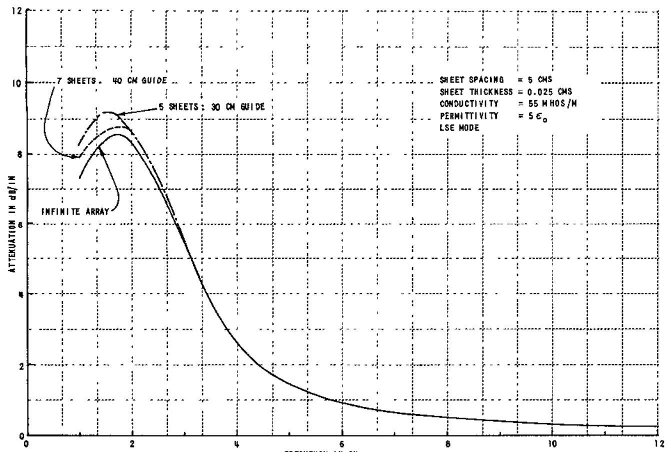
(c)

Fig. 4.

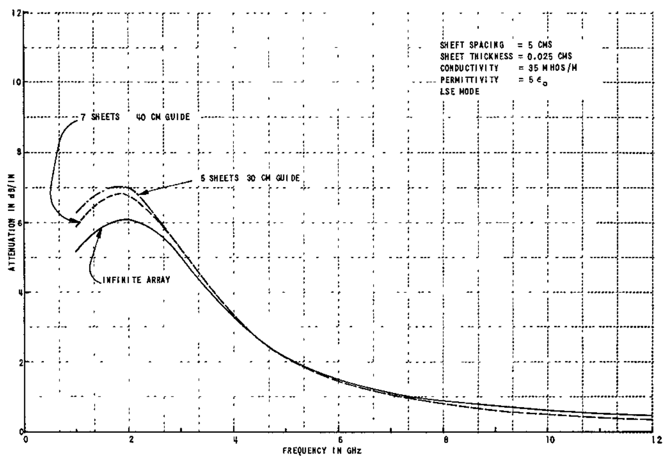
Suetake and Griemsmann derived an approximate expression for the peak attenuation

$$\alpha_{\max}=\frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}\,\frac{\delta}{c}$$

for the infinite array of sheets. Note that $\alpha_{\max}=\frac{1}{2}\sigma_{\text{average}}\sqrt{\mu/\epsilon}$ and that this is of the same form as α_∞, the high frequency asymptote for a completely filled waveguide. The



(a)



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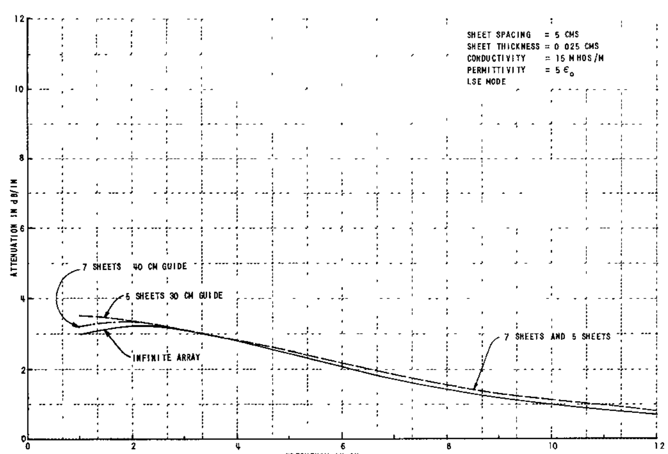


(c)

Fig. 5.

attenuation for a completely filled waveguide is discussed in Appendix B. This approximation is fairly good for the parameter values of interest and it applies in the waveguide calculation as well as for an infinite array. The frequency at which the peaks occur bears an inverse relationship to the sheet spacing; that is,

$$f_{\max} \, \alpha \, \frac{1}{c} \, .$$

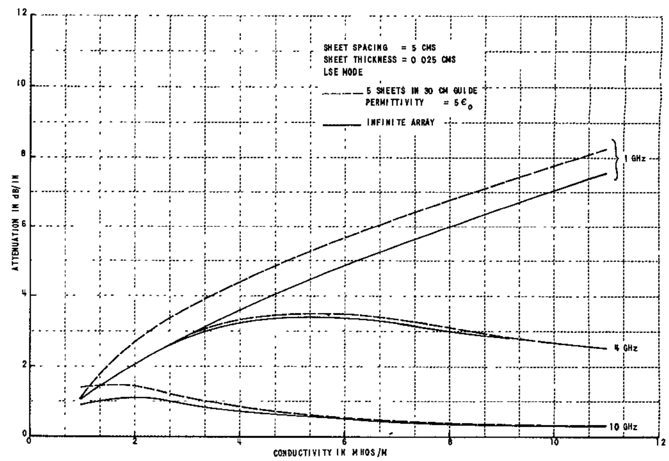


Fig. 6.

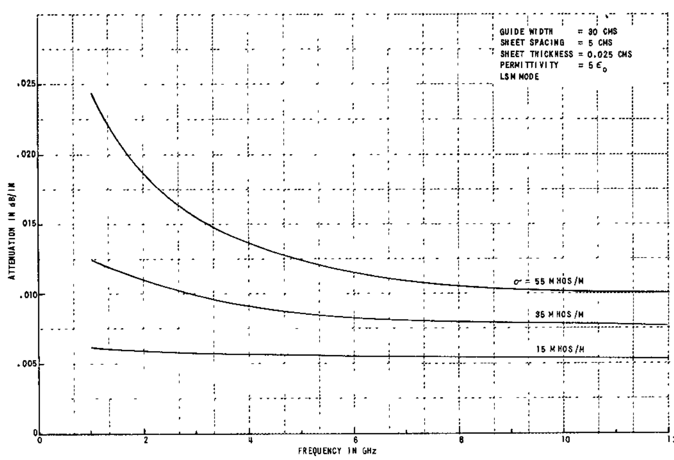


Fig. 7.

The value $f_{\max}$ depends upon conductivity, however, for conductivities in the range from 5 to 55 mhos/m $f_{\max}$ is very nearly fixed.

A waveguide completely filled with lossy material has an attenuation constant that increases monotonically with conductivity at a given frequency. For intermediate frequencies and large conductivity

$$\alpha \rightarrow \sqrt{\frac{\omega\mu\sigma}{2}} = \frac{1}{\text{skin depth}}$$

which is the usual approximation for good conductors. Figure 6 indicates that this increase occurs in waveguides with resistive sheets only at the lower frequencies. This is reasonable, however, since as conductivity increases the sheets become very good conductors and tend to decouple the fields between sheets. This is equivalent to each region between sheets acting as a separate waveguide.

Figure 7 shows attenuation versus frequency curves for the lowest-order LSM mode. Comparison with Fig. 4(a), (b), and (c) reveals that the attenuation is much less than for the lowest-order LSE mode. This is attributed in part to the fact that the electric field for the LSM mode has a component perpendicular to the sheets and the sheets have little effect upon it.

V. Conclusions

The propagation constants for a wave traveling down a waveguide with uniformly spaced conductive sheets having a finite width were found using an iterative computer technique. The results were shown to be quite similar, when several sheets are present and $\delta/c \ll 1$, to that obtained for an infinite array of parallel resistive sheets in free space. The infinite array case assumes thin sheets having a surface resistance. This case can be solved by computer or conformal mapping techniques. The general shape of the attenuation response can be determined through the solution of several relatively simple equations established through conformal mapping. The presence of metal walls around the lossy sheets have very little effect on the attenuation when five or more sheets are present. The insensitiveness of the side walls in modifying the attenuation due to the presence of the sheets means that the lossy materials can be placed at the most advantageous physical position.

Appendix A

Infinite Array of Parallel Resistive Sheets

Suetake and Griemsmann [1] considered microwave absorption of a plane wave incident on an infinite array of parallel resistive sheets of infinite extent. They assumed that the sheets are sufficiently thin that they can be completely represented by an ohms-per-unit square value R_s. The modes considered are those having the direction of propagation and the electric field vector parallel to the sheets, and for which the center planes of the resistive sheets are planes of infinite impedance. The dispersion equation obtained was

$$-j\frac{k_x c}{2}\tan\frac{k_x c}{2} = \frac{kc}{4}\frac{\eta}{R_s}$$

where

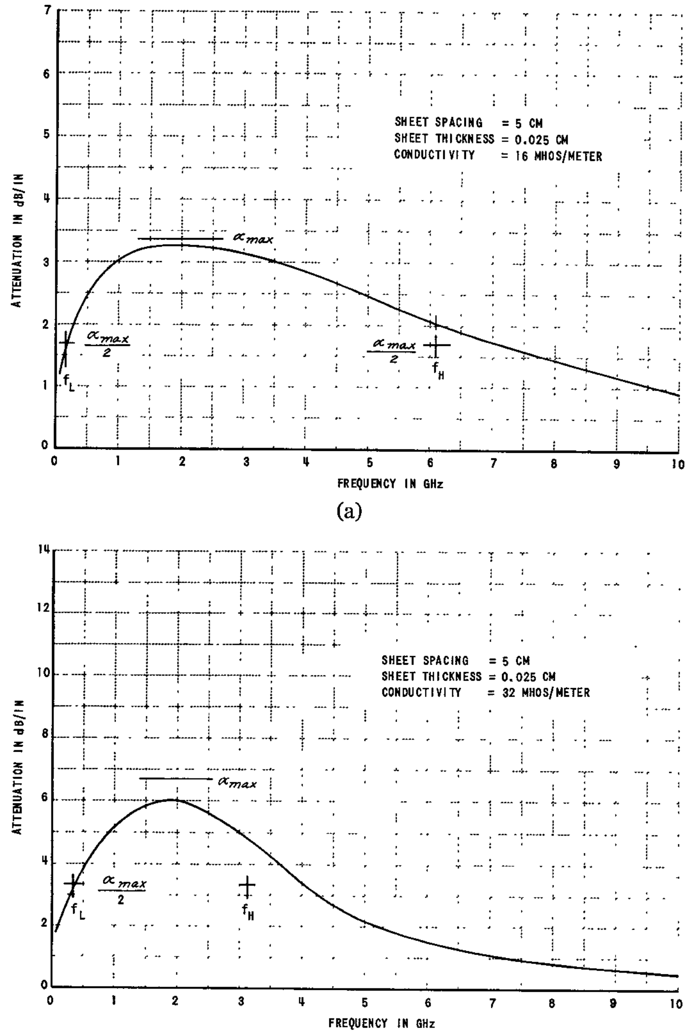
c is the spacing between sheets.
R_s is ohms-per-unit square value of the resistive sheets.
$\eta = \sqrt{\mu/\epsilon}$, $k = \omega\sqrt{\mu\epsilon}$ evaluated between the sheets.
γ is the z-directed propagation constant.
jk_x is the x-directed propagation constant.

The finite thickness sheets studied on this program convert to infinitely thin sheets by the relation $R_s = 1/\sigma\delta$. Suetake and Griemsmann concentrated on obtaining solutions by conformal mapping techniques, whereas in this project more accurate solutions were obtained by numerical methods using a digital computer.

The maximum value of attenuation for an infinite array of resistive sheets, as determined by conformal mapping techniques is

$$\alpha_{\max} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}\,\frac{\delta}{c}.$$

While the frequency for the maximum attenuation cannot be determined, it is possible to find the frequencies where $\alpha_{\max}$ is decreased to half its value. The frequencies where $\alpha = \frac{1}{2}\alpha_{\max}$ are given by



(a)

(b)

Fig. 8.

$$f_L = \frac{1}{8\pi\sqrt{\mu\epsilon}}\left[\frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}\,\frac{\delta}{c}\right]$$

and

$$f_H = \frac{\pi}{4\sqrt{\mu\epsilon}\,c^2}\left[\frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}\,\frac{\delta}{c}\right]^{-1}.$$

The computer solutions of the infinite array of parallel resistance sheets provided an opportunity to verify the accuracy of the approximate solutions obtained through conformal mapping technniqes. The comparison is illustrated by examples in Fig. 8(a) and (b). The condition for f_H to be valid, based on the conformal mapping techniques is

$$\alpha_{\max} < \frac{\pi}{\sqrt{2}\,c}.$$

The agreement between the results found through conformal mapping techniques and on the computer are quite good below the limit cited. As the conductivity is decreased, the high-frequency value for $\alpha_{\max}/2$ becomes too low in value. This disagreement becomes even more pronounced at lower values of conductivity.

Appendix B

Attenuation in a Waveguide Completely Filled with Lossy Material

Another limiting case of the parallel sheets having finite conductivity in a waveguide is a guide completely filled with lossy dielectric material. Since the case considered is for sheets of variable thickness, only a finite number of sheets are required to fill the guide with lossy material. An explicit expression for the attenuation constant of the filled guide is easily obtained and this limiting case result permits deduction of trends for a guide containing several sheets as parameter values are changed.

The attenuation for a waveguide filled with material of conductivity σ is

$$\alpha = \frac{1}{\sqrt{2}}\sqrt{\sqrt{(k^2 - k_c^2)^2 + k^2\frac{\sigma^2\mu}{\epsilon}} - (k^2 - k_c^2)}$$

where k_c is the cutoff wavenumber for the appropriate mode of the equivalent lossless guide, and $k^2 = \omega^2\mu\epsilon$. This expression is valid for all frequencies and it shows some interesting features for some limiting cases.

a) As $k \to 0$ the attenuation approaches the limiting value $\alpha_0 = k_c$.

b) As $k \to \infty$ the attenuation approaches the limiting value $\alpha_\infty = \frac{1}{2}\sigma\sqrt{\mu/\epsilon}$.

c) At $k = k_c$, the cutoff wavenumber for the equivalent lossless guide, $\alpha_{c0} = \sqrt{\alpha_0\alpha_\infty}$; that is, the geometric mean of the low- and high-frequency asymptotic values.

d) For $k_c = \frac{1}{2}\sigma\sqrt{\mu/\epsilon}$ the attenuation $\alpha = k_c = \frac{1}{2}\sigma\sqrt{\mu/\epsilon}$ is constant over the entire spectrum.

Figure 9 depicts the general form of the attenuation constant as a function of frequency. The three solutions for the attenuation in the waveguide show widely varying values

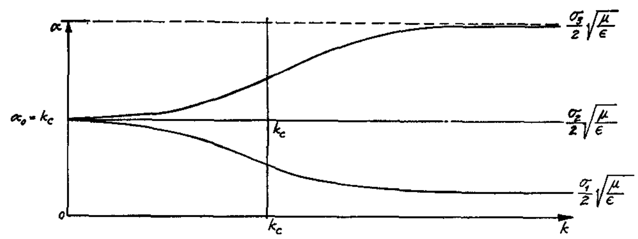


Fig. 9. Attenuation in lossy homogeneous waveguide.

depending on the relationship between k_c and $\frac{1}{2}\sigma\sqrt{\mu/\epsilon}$.

a) As noted previously, when $k_c = \frac{1}{2}\sigma\sqrt{\mu/\epsilon}$ the attenuation is constant over the entire frequency spectrum.

b) If $\frac{1}{2}\sigma 2\sqrt{\mu/\epsilon} > k_c$, the attenuation monotonically increases with frequency and asymptotically approaches $\frac{1}{2}\sigma\sqrt{\mu/\epsilon}$.

c) If $\frac{1}{2}\sigma\sqrt{\mu/\epsilon} < k_c$, the attenuation monotonically decreases with frequency and asymptotically approaches $\frac{1}{2}\sigma\sqrt{\mu/\epsilon}$.

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